

## EXTRA PRACTICE PROBLEMS FOR THE MIDTERM

1. Find  $\lim_{r \rightarrow \infty} \frac{2r^2 + 3r - 4}{5 - 3r^2}$  analytically.
2. Find  $\lim_{x \rightarrow 1} \frac{x^3 - 7x^2 + 12x}{4 - x}$ .
3. Find  $\lim_{x \rightarrow \infty} \frac{x^3 - 7x^2 + 12x}{4 - x}$ .
4. Is  $g$  continuous at  $x = 4$  if  $g(x) = \begin{cases} \frac{x^2 - 16}{x - 4} & x \neq 4 \\ 8 & x = 4 \end{cases}$ ?
5. Find  $\lim_{x \rightarrow a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a}$ .
6. Use the definition of the derivative to compute the  $f'(9)$  if  $f(x) = \sqrt{x}$ .
7. Use the definition of the derivative to compute  $f'(x)$  if  $f(x) = 3x^2 - x$ .
8. Find the tangent line to the curve  $y = \sin x$  when  $x = \pi/4$ .
9. Find the tangent line to the curve  $y = (3t^2 + t)^3$  when  $t = 1$ .
10. Find the normal line to the curve  $y = 3x^2 - 2x + 1$  when  $x = -1$ .
11. Compute  $\frac{d}{dx} \frac{e^{3x} - 24x^2}{x + 1}$ .
12. Compute  $\frac{d}{d\theta} \{ \sin^3 \theta + \tan^2(3\theta) \}$ .
13. Find  $\frac{d}{dr} (r^3 \sin r)$ .
14. Compute  $\lim_{s \rightarrow 1} \left( \frac{1}{s^2 - s} - \frac{1}{s - 1} \right)$ .
15. Find  $f''(x)$  if  $f(x) = \cos(\ln x)$ .
16. Find  $\frac{d^3 y}{dx^3}$  if  $y = 12x^5 - 3x^4 + 6x^3 - 2x^2 + 15$ .
17. Find  $\frac{dy}{dx}$  if  $x + y = \sin y$ .
18. Find  $\frac{d^2 y}{dx^2}$  if  $e^y + x = \tan(x + y)$ .
19. Find the tangent line to the curve  $x^2 + y^2 = 1$  at the point  $(3/5, 4/5)$ .
20. Find  $f'(3)$  if  $f(x) = \tan^2(\pi x/12)$ .
21. Find  $f'(2)$  if  $f(x) = 2^{3^x}$ .
22. Two stones are thrown vertically upward with matching initial velocities of 48 ft/s at time  $t = 0$ . One stone is thrown from the edge of a bridge that is 32 ft above the ground and the other stown is thrown from ground level. The height of the stone through from the bridge after  $t$  seconds is  $f(t) = -16t^2 + 48t + 32$  and the height of the stone thrown from the ground after  $t$  seconds is  $g(t) = -16t^2 + 48t$ .
  - a. Show that the stones reach their high points at the same time.
  - b. How much higher does the stone thrown from the bridge go than the stone thrown from the ground?
  - c. When do the stones strike the ground and with what velocities?
23. Compute  $\frac{d}{dt} \ln(\arctan t)$ .
24. Compute  $\frac{d}{dt} e^{\arcsin t}$ .
25. If  $f(x) = \arccos(x)$ , then find  $f'(1/2)$ .
26. Prove that the function  $f(x) = -x^3 - 3x^2 + 100$  has at least one zero on the real line.

27. Compute  $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\sin \theta}$
28. Compute  $\frac{d}{dx} \operatorname{arccsc} x$ .
29. Use the graph of  $g$  in the figure to do the following.
- (a) Find the values of  $x$  in  $(0, 7)$  at which  $g$  is not continuous.
  - (b) Find the values of  $x$  in  $(0, 7)$  at which  $g$  is not differentiable.
  - (c) Sketch a graph of  $g'$  on  $(0, 7)$ .

