MATH 2554 : Midterm Review

Nifty rules

Derivation

1. $\frac{d}{dx}c = 0$
2. $\frac{d}{dx}f(x) + g(x) = f'(x) + g'(x)$
3. $\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
4. $\frac{d}{dx}x^n = xn^{n-1}$
5. $\frac{d}{dx}cf(x) = cf'(x)$
6. $\frac{d}{dx}f(x) - g(x) = f'(x) - g'(x)$
7. $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$
8. $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$

The above show the following rules : constant rule (1), constant multiple rule (5), sum rule (2 & 6), product rule (3), quotient rule (7), power rule (4), chain rule (8)

Limit of a Function : Suppose the function f is defined for all x near a except possibly at a. If $f(x)$ is arbitrarily close to L (as close to L as we like) for all x sufficiently close (but not equal) to a we say :

$$\lim_{x \to a} f(x) = L$$

Continuity Checklist : A function f will be continuous at a if $\lim_{x \to a} f(x) = f(a)$, which can be expanded to the following checklist which should be followed in order to determine continuity :

1. $f(a)$ is defined (a is in domain of f)
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Intermediate Value Theorem : Suppose f is continuous on the interval $[a, b]$ and L is a number strictly between $f(a)$ and $f(b)$. Then there exists at least one number c in (a, b) satisfying $f(c) = L$

Derivative of a Function at a Point :

1. $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
2. $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Definition of the Derivative :

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Random Tips

1. $\lim_{x \to -a} f(x) \not\equiv \lim_{x \to a^-} f(x)$ as a^- implies a left-sided limit, don't make this simple mistake!
2. To follow correct limit notation, <u>do not</u> plug in your values with your limit sign still attached, e.g. $\lim_{x \to 5} 3x + 5 = 3(5) + 5 = 20$ NOT $\lim_{x \to 5} 3x + 5 = \lim_{x \to 5} 3(5) + 5 = 20$
3. Remember that **vertical asymptotes** $x = a$ occur when $\lim_{x \to a} f(x) = \pm\infty$, $\lim_{x \to a^-} f(x) = \pm\infty$, or $\lim_{x \to a^+} f(x) = \pm\infty$ while a **horizontal asymptote** $y = L$ occurs at $\lim_{x \to -\infty} f(x) = L$ or $\lim_{x \to \infty} f(x) = L$
4. Just as $y = x$ gives the derivation $y' = 1$, with the chain rule we can use **implicit differentiation** to find $y^2 = x$ gives $2y \cdot y' = 1$ which reduces to $y' = \frac{1}{2y}$. To find the second derivative, simply repeat and replace any y' with your first answer. Using the previous answer this gives $y'' = -\frac{1}{2}y^{-2} \cdot y' = -\frac{1}{2}y^{-2} \cdot \frac{1}{2y} = -\frac{1}{4y^3}$
5. **Speed** is simply the absolute value of **velocity** ($|v(t)| =$ speed). An object will **hit the ground** when the position function $s(t) = 0$, while an object will reach it's **highest point** when the tangent line of $s(t)$ is horizontal, that is $s'(t) = v(t) = 0$

Basic derivative forms

Trig derivatives :

1. $\dfrac{d}{dx}\sin x = \cos x$
2. $\dfrac{d}{dx}\cos x = -\sin x$
3. $\dfrac{d}{dx}\tan x = \sec^2 x$
4. $\dfrac{d}{dx}\cot x = -\csc^2 x$
5. $\dfrac{d}{dx}\sec x = \sec x \tan x$
6. $\dfrac{d}{dx}\csc x = -\csc x \cot x$

Exponential/Log derivatives :

1. $\dfrac{d}{dx}e^x = e^x$
2. $\dfrac{d}{dx}\ln|x| = \dfrac{1}{x}$
3. $\dfrac{d}{dx}b^x = b^x \ln b$
4. $\dfrac{d}{dx}\log_b|x| = \dfrac{1}{x \ln b}$