EXTRA PRACTICE PROBLEMS FOR EXAM 2

1. Find the intervals on which the function $f(x) = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{15}{2}x^2 + 8$ is increasing or decreasing.
2. Sketch a graph of a function f that is continuous on $(-\infty, \infty)$ so that: $f'(0)$ is undefined, f is increasing on $(-\infty, 0)$, f is increasing on $(3, \infty)$, f is decreasing on $(0, 3)$, $\lim_{x\to-\infty} f(x) = -3$, and $\lim_{x\to\infty} f(x) = \infty$.
3. Sketch a graph of a function f tha is continuous on $(-\infty, \infty)$ so that: $f'(-4) = f'(-3) = f'(0) = f'(4) = f'(6) = 0$, f has local maxima at $x = -3$ and $x = 6$ and local minima at $x = -4$ and $x = 4$ and f has no local extrema at $x = 0$.
4. Find the critical points of the functions: a) $f(x) = 2x^3 + 3x^2 - 12x + 1$ on $[-2, 4]$ and b) $f(x) = \frac{x^2}{x^2+1}$ on $[-4, 4]$. Then b) Use the First Derivative Test to local the local maximum and minimum values. Finally, c) Find the absolute extrema on the given intervals.
5. Do a complete curve sketch of the functions $f(x) = 3x - x^3$ and $g(x) = \frac{x^2}{x^2-4}$. Your curve sketch should incorporate the local extrema, inflection points, x- and y- intercepts (when they exist) as well as regions on which f is increasing/decreasing and concave up/down.
6. Sketch a curve of a function f that is continuous on $(-\infty, \infty)$ and satisfies all of the following properties:
   - $f' < 0$ and $f" < 0$ for $x < -1$
   - $f' < 0$ and $f" > 0$ for $-1 < x < 2$
   - $f' > 0$ and $f" > 0$ for $2 < x < 8$
   - $f' > 0$ and $f" < 0$ for $8 < x < 10$
   - $f' > 0$ and $f" > 0$ for $x > 10$.
7. A piece of wire of length 60 is cut, and the resulting two pieces are formed to make a circle and a square. Where should the wire be cut to (a) minimize and (b) maximize the combined area of the circle and the square.
8. Find numbers x and y satisfying the equation $xy = 12$ such that the sum $2x + y$ is as small as possible.
9. Find the point P on the line $y = 2x$ that is closest to the point $(10, 12)$.
10. Approximate the change in the atmospheric pressure when the altitude increases from $z = 2$km to $z = 2.01$km where the atmospheric pressure is given by $P(z) = 1000e^{-z/10}$.
11. Apprixmate the change in the area of the square when its side length changes from $s = 2$m to $s = 2.02$m.
12. Find all intervals where the function $x^2 - 9$ is increasing/decreasing.
13. Find all intervals where the function $x^2 + \ln x^2 - x$ is increasing/decreasing.
14. Find all intervals where $3x \ln x - 2x^2$ is concave up/down.
15. For the function $f(x) = -x^4 - 2x^3 + 12x^2$, find all intervals where the function is increasing/decreasing, concave up/down, and all extrema.
16. Find all critical points $-x^3 + 9x$ on $[-4, 3]$. Identify all local and global extrema.
17. Find all the critical points of $\arctan x - x^3$ on $[-1, 1]$.
18. Find the maximal volume of a square based box with the sum of its length, width, and height 39 cm.
19. What is the rectangle of maximal area constructed with its base on the diameter of a semicircle of radius 14 m with its vertices on the semicircle.
20. Find the maximal volume of an open box formed by removing square sections from a rectangle with side lengths 10 and 16.
21. A farmer needs to build a fence out of 300 m of fencing so that it abuts his barn and has three divisions. What is the maximal area he can enclose?
22. Write the linear approximation of $\sin x$ at $\pi/4$. Use it to approximate $\sin(0.75)$.
23. Give an approximation of $\sqrt{146}$.
24. Determine if the following limits have an indeterminant form, state it, and use L'Hopital's rule to compute the limit if possible.
   (a) $\lim_{x\to1} \frac{x^4+3x^2-1}{x-3}$,
   (b) $\lim_{x\to0} \frac{2\sin(3x)}{5x}$,
   (c) $\lim_{x\to\infty} \frac{x^2-\ln(2/x)}{x^2-5x}$,
   (d) $\lim_{x\to0} \frac{3x^2+\sin(e^x)}{x-2}$,
   (e) $\lim_{x\to0}(1-x)\tan(\frac{\pi x}{2})$,
   (f) $\lim_{x\to\infty}(1+\frac{2}{x^2})^{x^2}$.