Upcoming stuff

- Thursday December 6: Quiz 16 (in drill)
- Sunday December 9: Computer HW 15 due
- Monday December 10: Final Exam, 5:30 - 7:30 pm, Location JBHT 144
- Fill out course evaluations! Right now this is at 50%.

5.5 – Substitution Rule

MATH 2554 – Calculus I

Substitution Rule.

We have seen a few methods to find antiderivatives (e.g., power rule, knowledge of derivatives, etc.). However, for many functions, it is more challenging to find the antiderivative.

Today we examine the substitution rule as a method to integrate.

Integration by Trial and Error: Integration by Trial and Error, aka, Guess and Check is a relatively inefficient method.

Exercise: $\int \cos(2x + 5)\, dx$.

Techniques of Integration: Each derivative rule (chain rule, product rule, etc.) gives rise to a technique of integration. This topic gets fully explored in Calc II. For now, we use the chain rule to establish the integration technique known as Integration by Substitution.

Recall:

$$\frac{d}{dx}F\big(g(x)\big) = F'\big(g(x)\big)g'(x).$$

Anti-differentiating both sides yields

$$F\big(g(x)\big) + C = \int F'\big(g(x)\big)g'(x)\,dx.$$

If the inner function is $u = g(x)$ and we write $F'(x) = f(x)$, then $du = g'(x)\,dx$, and we see that

$$\int f\big(g(x)\big)g'(x)\,dx = \int f(u)\,du.$$

Theorem (Substitution Rule for Indefinite Integrals)

Let $u = g(x)$, where g' is continuous on an interval, and let f be continuous on the corresponding range of g. On that interval,

$$\int f\big(g(x)\big)g'(x)\,dx = \int f(u)\,du.$$

Procedure

1. Given an indefinite integral involving a composite function $f(g(x))$, identify an inner function $u = g(x)$ such that a constant multiple of $g'(x)$ appears in the integrand.
2. Substitute $u = g(x)$ and $du = g'(x)\,dx$ in the integral.
3. Evaluate the new indefinite integral with respect to u.
4. Write the result in terms of x using $u = g(x)$.

NOTE: Not all integrals yield to the Substitution Rule.

Compute:

1. $\int 8x \cos(4x^2 + 3)\, dx$
2. $\int \sin^{10}\theta \cos\theta\, d\theta$
3. $-\int \dfrac{\csc r \cot r}{1 + \csc r}\, dr$
4. $\int \dfrac{1}{10x - 3}\, dx$
5. $\int (3x^2 + 8x + 5)^8 (3x + 4)\, dx.$

Sneakier Substitutions:

1. $\displaystyle\int \frac{y^2}{y+1}\,dy$
2. $\displaystyle\int \frac{s}{\sqrt{s+1}}\,ds.$

Substitution Rules for Definite Integrals. There are two methods for computing definite integrals via substitution.

Theorem (Substitution Rule for Definite Integrals – The Good Method)

Let $u = g(x)$, where g' is continuous on $[a, b]$, and let f be continuous on the range of g. Then

$$\int_a^b f\big(g(x)\big)g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

Examples:

1. $\displaystyle\int_0^4 \frac{1}{(4x-3)^4}\,dx$
2. $\displaystyle\int_0^{\ln 4} \frac{e^x}{1+2e^x}\,dx$

More examples!

1. $\displaystyle\int_1^e \frac{\ln x}{x}\,dx$
2. $\displaystyle\int_0^{\pi/3} \frac{\sin x}{\cos x}\,dx$

Homework Problems: 17-73, 79-85 odd