4 more days!

- Thursday November 29: Quiz 15 (take home) given in drill
- Sunday December 2: Computer HW 14 due
- Tuesday December 4: Quiz 15 due in drill (OPEN)
- Thursday December 6: Quiz 16 (in drill)
- Sunday December 9: Computer HW 15 due
- Monday December 10: Final Exam, 5:30 - 7:30 pm, Location JBHT 144
- Final exam study guide posted

5.4 – Working with Integrals

MATH 2554 – Calculus I

Applications of the Integral: There are hundreds if not thousands of applications of the integral. Chapter 6 is devoted solely to applications of the integral which you will see in Calculus II. In this section, we start to build some tools that assist in applications.

We focus on three specific tools:

1. Integration of even and odd functions.
2. Finding the average value of a function.
3. The Mean Value Theorem for Integrals.

Recall that a function f is odd if

$$f(x) = -f(-x)$$

and f is even if

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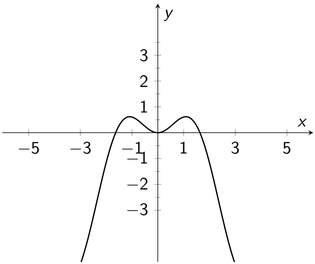
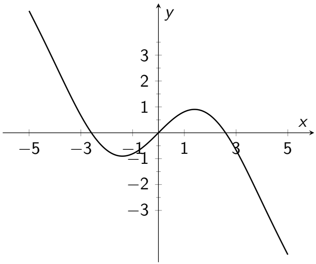


even



odd

By symmetry,

Theorem (Integrals of Even and Odd Functions)

Let $a > 0$ and f be an integrable function on the interval $[-a, a]$.

1. *If f is even, then* $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.
2. *If f is odd, then* $\int_{-a}^{a} f(x)\,dx = 0$.

Exercises: Find

1. $\displaystyle\int_{-4}^{4} 3x^2 - x \, dx$
2. $\displaystyle\int_{-1}^{1} 1 - |x| \, dx$
3. $\displaystyle\int_{-\pi}^{\pi} \sin^3 \theta \, d\theta.$

Average Value of a Function. The average value of a function is computed in a similar manner as finding the average of a set of numbers.

Let f be a function on an interval $[a, b]$. Partition the interval into n even subintervals and choose a point x_k^* in each of the subintervals. Then an approximation for the average of f is

$$\frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{n}.$$

Since $n\Delta x = b - a$, it follows that $n = \dfrac{b-a}{\Delta x}$. This means

$$\begin{aligned}\frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{n} &= \frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{\frac{b-a}{\Delta x}} \\ &= \frac{1}{b-a}\left(f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)\right)\Delta x.\end{aligned}$$

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Letting $n \to \infty$, we obtain the average value of f.

Definition (Average Value of a Function)

The average value of an integral function f on the interval $[a, b]$ is

$$\bar{f} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

Example: Find the average value of $f(x) = x^2(1 - x)$ on $[-2, 3]$.

Theorem (Mean Value Theorem for Integrals)

Let f be continuous on the interval $[a, b]$. There exists a point c in (a, b) such that

$$f(c) = \bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Exercise: Find or approximate the point(s) at which the function $f(x) = x^2 - 2x + 1$ equals its average value on the interval $[0, 2]$.

Homework Problems: 11-43 odd (pgs. 385-386).