The beginning of the end!

- Tuesday November 27: Quiz 14 (take home) due
- Thursday November 29: Quiz 15 (take home) given in drill
- Sunday December 2: Computer HW 14 due
- Tuesday December 4: Quiz 15 due in drill
- Thursday December 6: Quiz 16 (in drill)
- Sunday December 9: Computer HW 15 due
- Monday December 10: Final Exam, 5:30 - 7:30 pm, Location TBD

5.3 – Fundamental Theorem of Calculus

MATH 2554 – Calculus I

Problem: Computing definite integrals via Riemann sums is laborious and impractical.

Question: Is there a better way?

Answer: Yes!!

Critical Tool: The area function!

Definition (Area Function)

Let f be a continuous function for $t \geq a$. The area function for f with left endpoint a is

$$A(x) = \int_a^x f(t)\,dt$$

where $x \geq a$.

The area function gives the net area of the region bounded by the graph of f and the t-axis on the interval $[a, x]$.

DRAW ILLUSTRATIVE PICTURES!

Exercise: Let $f(t) = 4t + 3$ and define $A(x) = \int_1^x f(t)\,dt$.

Find

1. $A(2)$
2. $A(5)$
3. $A(x)$
4. $A'(x)$

Theorem (Fundamental Theorem of Calculus (Part 1))

If f is continuous on $[a, b]$, then the area function

$$A(x) = \int_a^x f(t)\,dt, \quad \text{for } a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b). The area function satisfies $A'(x) = f(x)$. Equivalently,

$$A'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

which means that the area function of f is an antiderivative of f on $[a, b]$.

Theorem (Fundamental Theorem of Calculus (Part 2))

If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Compute

1. $\int_1^2 (4t + 3)\, dt$
2. $\int_0^1 (1 - e^x)\, dx$
3. $\int_1^y f'(s)\, ds$
4. $\frac{d}{dx} \int_x^{10} \frac{dz}{z^2 + 1}$
5. $\frac{d}{dx} \int_x^{x^2} t^2 - 4t + 1\, dt$

Homework Problems: Do problems 13-85 odd (pgs. 378-379).