5.3-5.5

5.3 - Fundamental Theorem of Calculus

What is the Fundamental Theorem?

Fundamental Theorem of Calculus : If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$(1) \quad F(x) = \int_a^x f(t)dt$$

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Another way of writing (1):

$$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$$

Deriving Antiderivatives

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\frac{d}{dx}\int_x^0 \frac{ds}{\sqrt{s^2+1}}$$

Deriving Antiderivatives

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

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$$\frac{d}{dx}\int_x^0 \frac{ds}{\sqrt{s^2+1}} = \frac{d}{dx}[F(0) - F(x)] = -f(x) = -\frac{1}{\sqrt{x^2+1}}$$

Deriving Antiderivatives

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F(g(x)) if g(x) = x^2

Definite Integrals

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\int_0^1 \frac{1}{e^x} = \int_0^1 e^{-x} = -e^{-x}\Big|_0^1 = -e^{-1} - (-e^0) = -\frac{1}{e} + 1$$

5.4 - Working with Integrals

Skipping symmetry - worth looking into though! (Dec 5th Prob 1)

Average Value of a Function

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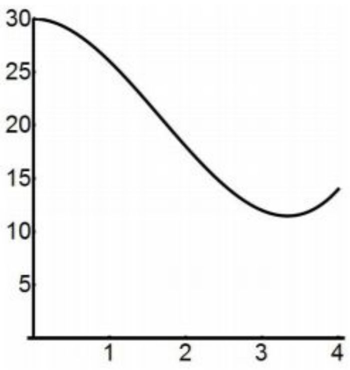
Average Value of a Function

(3) The elevation of a path is given by $f(x) = x^3 - 5x^2 + 30$ feet above sea level, where x measures horizontal distance in miles. Find the average value of the elevation function for $0 \leq x \leq 4$ and indicate it on the graph.



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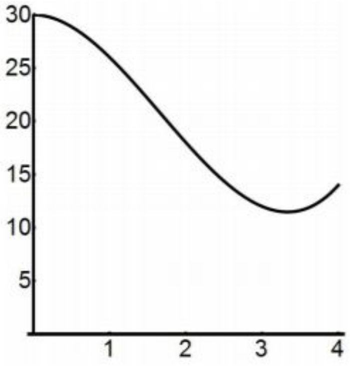

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)dx$$

$$\bar{f} = \frac{1}{4}\int_0^4 x^3 - 5x^2 + 30 = \frac{1}{4}\left(\frac{x^4}{4} - \frac{5x^3}{3} + 30x\right)\Big|_0^4 = \frac{232}{12} = 19.\bar{3}$$

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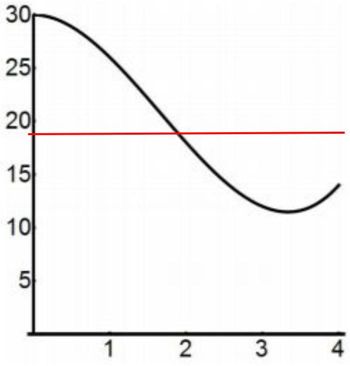

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5.5 - Substitution

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$$\frac{1}{2}\int_9^{25} \frac{1}{\sqrt{u}}du = \frac{1}{2}\int_9^{25} u^{-1/2}du = \frac{1}{2}(2 \cdot u^{1/2})\Big|_9^{25} = \sqrt{25} - \sqrt{9} = \boxed{2}$$