MATH 2554 : 3.11 Review Sheet

Key Concepts

I'm sorry for the the wall of text... I promise it won't happen again ?

Related Rates : Students often find this section especially daunting, so I've complied some personal tips in addition to the books recommended procedure. The following steps should be taken :

1. **Sketch the problem** and write down every given value
2. **Identify the "rate"** (derivative !) you are trying to find
3. **Write down the equation** which explains the relationship between the variable for which the rate is given and the variable which you need to find the rate of
4. **Substitute known values or functions**... the function aspect of this will be explained more below !
5. **Check your units !**

In related rates problems, you will often encounter examples which contain multiple variables. However, **if the right side of your equation contains multiple variables, they must be redefined in terms of the variable you are solving or replaced with a numerical value.** If you are solving the rate of the left side of the equation, you will instead replace the variables such that you only have the variable for which the rate is given.

If you have multiple units in the right side of your equation, you'll either have one of two scenarios :

1. If the variable you're replacing is **constant over time**, just plug in the **numerical value** given for the variable
2. If the variable **changes over time**, you'll need to plug in a relationship between that variable and the other. This will behave as a **function.**

Examples :

1. Say we want to find rate at which water level $h'(t)$ is increasing for a cylinder with a height h of 5, radius r of 3, and rate of change of volume $V'(t)$. The equation expressing the relationship in this scenario will be $V(t) = \pi r^2 h$. In this case, radius r is constant so we can simply replace $r^2 = 3^2 = 9$ giving us $V(t) = 9\pi h$. To find $h'(t)$ we simple derive $V(t)$ and find $V'(t) = 9\pi h'(t) \to h'(t) = V'(t)/(9\pi)$ and plug in the $V'(t)$ given.
2. Say instead we want to find the rate at which water level $h'(t)$ is increasing at some given height h in a cone with the same height and radius. (Notice some height had to be given in addition to $V'(t)$ **Notice both radius and height are changing over time**, so instead we must find a relationship between r and h. For a cone $V(t) = \frac{1}{3}\pi r^2$. Due to the rule of similar triangles, if the initial height h is 5 and initial radius r is 3, the ratio between the two will remain $\frac{r}{h} = \frac{3}{5}$ so we can say $r = \frac{3}{5}h$ and derive $V(t) = \frac{1}{3}\pi(\frac{3}{5}h)^2 h \to V(t) = \frac{3}{25}\pi h(t)^3$. So then, $V'(t) = \frac{9}{25}\pi h^2 \cdot h'(t)$ so $h'(t) = 25V'(t)/(9\pi h^2)$.

Okay, I know those examples were gross to try to explain on a sheet. If you're having a problem with this section, don't hesitate to email me and ask for help or specific examples of these two "scenarios".